Module 4

Game Theory

Learning Objectives

After completing this supplement, students will be able to:

1. Understand the principles of zero-sum, two-person games.
2. Analyze pure strategy games and use dominance to reduce the size of a game.
3. Solve mixed strategy games when there is no saddle point.

Supplement Outline

M4.1 Introduction
M4.2 Language of Games
M4.3 The Minimax Criterion
M4.4 Pure Strategy Games
M4.5 Mixed Strategy Games
M4.6 Dominance

Summary • Glossary • Solved Problems • Self-Test • Discussion Questions and Problems • Bibliography

Appendix M4.1: Game Theory with QM for Windows

M4.1 INTRODUCTION

As discussed in Chapter 1, competition can be an important decision-making factor. The strategies taken by other organizations or individuals can dramatically affect the outcome of our decisions. In the automobile industry, for example, the strategies of competitors to introduce certain models with certain features can dramatically affect the profitability of other carmakers. Today, business cannot make important decisions without considering what other organizations or individuals are doing or might do.

Game theory is one way to consider the impact of the strategies of others on our strategies and outcomes. A *game* is a contest involving two or more decision makers, each of whom wants to win. *Game theory* is the study of how optimal strategies are formulated in conflict.

The study of game theory dates back to 1944, when John von Neumann and Oscar Morgenstern published their classic book, *Theory of Games and Economic Behavior*.[1] Since then, game theory has been used by army generals to plan war strategies, by union negotiators and managers in collective bargaining, and by businesses of all types to determine the best strategies given a competitive business environment.

Game theory continues to be important today. In 1994, John Harsanui, John Nash, and Reinhard Selten jointly received the Nobel Prize in Economics from the Royal Swedish Academy of Sciences.[2] In their classic work, these individuals developed the notion of *noncooperative game theory*. After the work of John von Neumann, Nash developed the concepts of the Nash equilibrium and the Nash bargaining problem, which are the cornerstones of modern game theory.

In a zero-sum game, what is gained by one player is lost by the other.

Game models are classified by the *number of players*, the *sum of all payoffs*, and the *number of strategies* employed. Owing to the mathematical complexity of game theory, we limit the analysis in this module to games that are two person and zero sum. A *two-person game* is one in which only two parties can play—as in the case of a union and a company in a bargaining session. For simplicity, X and Y represent the two game players. *Zero sum* means that the sum of losses for one player must equal the sum of gains for the other player. Thus, if X wins 20 points or dollars, Y loses 20 points or dollars. With any zero-sum game, the sum of the gains for one player is always equal to the sum of the losses for the other player. When you sum the gains and losses for both players, the result is zero—thus the name *zero-sum games.*

M4.2 LANGUAGE OF GAMES

To introduce the notation used in game theory, let us consider a simple game. Suppose there are only two lighting fixture stores, X and Y, in Urbana, Illinois. (This is called a *duopoly*.) The respective market shares have been stable up until now, but the situation may change. The daughter of the owner of store X has just completed her MBA and has developed two distinct advertising strategies, one using radio spots and the other newspaper ads. Upon hearing this, the owner of store Y also proceeds to prepare radio and newspaper ads.

The 2×2 payoff matrix in Table M4.1 shows what will happen to current market shares if both stores begin advertising. By convention, payoffs are shown only for the first game player, X in this case. Y's payoffs will just be the negative of each number. For this game, there are only two strategies being used by each player. If store Y had a third strategy, we would be dealing with a 2×3 payoff matrix.

[1] J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton, NJ: Princeton University Press, 1944.

[2] Rita Koselka. "The Games Businesses Play," *Forbes* (November 7, 1994): 12.

TABLE M4.1

Store X's Payoff Matrix

		GAME PLAYER Y's STRATEGIES	
		Y_1 (Use radio)	Y_2 (Use newspaper)
GAME PLAYER X's STRATEGIES	X_1 (Use radio)	3	5
	X_2 (Use newspaper)	1	–2

A positive number in Table M4.1 means that X wins and Y loses. A negative number means that Y wins and X loses. It is obvious from the table that the game favors competitor X, since all values are positive except one. If the game had favored player Y, the values in the table would have been negative. In other words, the game in Table M4.1 is biased against Y. However, since Y must play the game, he or she will play to minimize total losses. To do this, Player Y would use the minimax criterion, our next topic.

Game Outcomes

STORE X's STRATEGY	STORE Y's STRATEGY	OUTCOME (% CHANGE IN MARKET SHARE)
X_1 (use radio)	Y_1 (use radio)	X wins 3 and Y loses 3
X_1 (use radio)	Y_2 (use newspaper)	X wins 5 and Y loses 5
X_2 (use newspaper)	Y_1 (use radio)	X wins 1 and Y loses 1
X_2 (use newspaper)	Y_2 (use newspaper)	X loses 2 and Y wins 2

M4.3 THE MINIMAX CRITERION

A player using the minimax criterion will select the strategy that minimizes the maximum possible loss.

For two-person, zero-sum games, there is a logical approach to finding the solution: In a zero-sum game, each person should choose the strategy that minimizes the maximum loss, called the *minimax criterion.* This is identical to maximizing one's minimum gains (so for one player this could be called the maximin criterion).

Let us use the example in Table M4.1 to illustrate the minimax criterion. This is a two-person zero-sum game with the strategies for player Y given as the columns of the table. The values are gains for player X and losses for player Y. Player Y is looking at a maximum loss of 3 if strategy Y_1 is selected and a maximum loss of 5 if strategy Y_2 is selected. Thus, Player Y should select strategy Y_1, which results in a maximum loss of 3 (the minimum of the maximum possible losses). This is called the *upper value of the game.* Table M4.2 illustrates this minimax approach.

The upper value of the game is equal to the minimum of the maximum values in the columns.

The lower value of the game is equal to the maximum of the minimum values in the rows.

In considering the maximin strategy for player X (whose strategies correspond to the rows of the table), let us look at the minimum payoff for each row. The payoffs are +3 for strategy X_1 and –2 for strategy X_2. The maximum of these minimums is +3, which means strategy X_1 will be selected. This value (+3) is called the *lower value of the game.*

If the upper and lower values of a game are the same, this number is called the *value of the game*, and an equilibrium or *saddle point* condition exists. For the game presented in

TABLE M4.2

Minimax Solution

	SADDLE POINT		
	Y_1	Y_2	**Minimum**
X_1	3	5	**3** ← ***Maximum of minimums***
X_2	1	–2	**–2**
Maximum	**3**	**5**	

Minimum of maximums

An equilibrium or saddle point condition exists if the upper value of the game is equal to the lower value of the game. This is called the value of the game.

Table M4.2, the value of the game is 3, because this is the value for both the upper and lower values. The value of the game is the average or expected game outcome if the game is played an infinite number of times.

In implementing the minimax strategy, player Y will find the maximum value in each column and select the minimum of these maximums. In implementing the maximin strategy, player X will find the minimum value in each row and select the maximum of these minimums. When a saddle point is present, this approach will result in pure strategies for each player. Otherwise, the solution to the game will involve mixed strategies. These concepts are discussed in the following sections.

M4.4 PURE STRATEGY GAMES

A pure strategy exists whenever a saddle point is present.

When a saddle point is present, the strategy each player should follow will always be the same regardless of the other player's strategy. This is called a *pure strategy*. A saddle point is a situation in which both players are facing pure strategies.

Using minimax criterion, we saw that the game in Table M4.2 had a saddle point and thus is an example of a pure strategy game. It is beneficial for player X and for player Y to always choose one strategy. Simple logic would lead us to this same conclusion. Player X will always select X_1, since the payoffs for X_1 are better than the payoffs for X_2 regardless of what player Y does. Knowing that player X will select X_1, player Y will always select strategy Y_1 and only lose 3 rather than 5. Note that the saddle point in this example, 3, is the largest number in its column and the smallest number in its row. This is true of all saddle points.

Another example of a pure strategy game is shown in Table M4.3. Notice that the value 6 is the lowest number in its row and the highest number in its column. Thus, it is a saddle point and indicates that strategy X_1 will be selected by player X and strategy Y_2 will be selected by player Y. The value of this game is 6.

TABLE M4.3

Example of a Pure Strategy Game

		PLAYER Y's STRATEGIES		
		Y_1	Y_2	*Minimum row number* ↓
PLAYER X's STRATEGIES	X_1	10	6	6
	X_2	–12	2	–12
Maximum column number →		10	6	

IN ACTION Game Theory in the Brewing Business

Companies that understand the principles and importance of game theory can often select the best competitive strategies. Those companies that don't can face financial loss or even bankruptcy. The successful and unsuccessful selection of competitive gaming strategies can be seen in most industries, including the brewing industry.

In the 1970s, Schlitz was the second-largest brewer in the United States. With its slogan "the beer that made Milwaukee famous," Schlitz was chasing after the leader in beer sales, Anheuser-Busch, maker of Budweiser. Schlitz could either keep its current production output or attempt to produce more beer to compete with Anheuser-Busch. It decided to get more beer to the market in a shorter amount of time. In order to accomplish this, Schlitz selected a strategy of distributing "immature" beer. The result was cloudy beer that often contained a slimy suspension. The beer and Schlitz's market share and profitability went down the drain. Anheuser-Busch, Miller, and Coors became the market leaders.

Similarly when Miller first decided to market Miller Lite, with the slogan "tastes great—less filling," Anheuser-Busch had two possible gaming strategies: to develop its own low-calorie beer or to criticize Miller in its advertising for producing a watered down beer. The strategy it selected was to criticize Miller in its advertising. The strategy didn't work, Miller gained significant market share, and Anheuser-Busch was forced to come out with its own low-calorie beer—Bud Light.

Today, Anheuser-Busch, Miller, Coors, and other large beer manufacturers face new games and new competitors that produce micro-brews, dry beer, and ice beer. Although it is too early to tell what the large beer makers will do and how successful their strategies will be, it appears that their strategy will be to duplicate what these smaller brewers are doing. What is clear, however, is that a knowledge of the fundamentals of game theory can make a big difference.

Source: Philip Van Munching. "American Brewing, Unreal," *The Economist* (September 6, 1997): 24.

M4.5 MIXED STRATEGY GAMES

In a mixed strategy game, each player should optimize the expected gain.

When there is no saddle point, players will play each strategy for a certain percentage of the time. This is called a *mixed strategy game*. The most common way to solve a mixed strategy game is to use the expected gain or loss approach. The goal of this approach is for a player to play each strategy a particular percentage of the time so that the expected value of the game does not depend upon what the opponent does. This will only occur if the expected value of each strategy is the same.

Consider the game shown in Table M4.4. There is no saddle point, so this will be a mixed strategy game. Player Y must determine the percentage of the time to play strategy Y_1 and the percentage of the time to play strategy Y_2. Let P be the percentage of time that player Y chooses strategy Y_1 and $1 - P$ be the percentage of time that player Y chooses strategy Y_2. We must weight the payoffs by these percentages to compute the expected gain for each of the different strategies that player X may choose.

For example, if player X chooses strategy X_1, then P percent of the time the payoff for Y will be 4, and $1 - P$ percent of the time the payoff will be 2, as shown in Table M4.5. Similarly, if player X chooses strategy X_2, then P percent of the time the payoff for Y will be 1, and $1 - P$ percent of the time the payoff will be 10. If these expected values are the same,

TABLE M4.4

Game Table for Mixed Strategy Game

		PLAYER Y's STRATEGIES	
		Y_1	Y_2
PLAYER X's STRATEGIES	X_1	4	2
	X_2	1	10

TABLE M4.5

Game Table for Mixed Strategy Game with Percentages (P, Q) Shown

		Y_1	Y_2	
		P	$1 - P$	**Expected gain**
X_1	Q	4	2	$4P + 2(1 - P)$
X_2	$1 - Q$	1	10	$1P + 10(1 - P)$
Expected gain		$4Q + 1(1 - Q)$	$2Q + 10(1 - Q)$	

then the expected value for player Y will not depend on the strategy chosen by X. Therefore, to solve this, we set these two expected values equal, as follows:

$$4P + 2(1 - P) = 1P + 10(1 - P)$$

Solving this for P we have

$$P = 8/11$$

and

$$1 - P = 1 - 8/11 = 3/11.$$

Thus, $8/11$ and $3/11$ indicate how often player Y will choose strategies Y_1 and Y_2 respectively. The expected value computed with these percentages is

$$1P + 10(1 - P) = 1(8/11) + 10(3/11) = 38/11 = 3.46.$$

Performing a similar analysis for player X, we let Q be the percentage of the time that strategy X_1 is played and $1 - Q$ be the percentage of the time that strategy X_2 is played. Using these, we compute the expected gain shown in Table M4.5. We set these equal, as follows:

$$4Q + 1(1 - Q) = 2Q + 10(1 - Q)$$

Solving for Q we get

$$Q = 9/11$$

and

$$1 - Q = 2/11.$$

Thus, $9/11$ and $2/11$ indicate how often player X will choose strategies X_1 and X_2 respectively. The expected gains with these probabilities will also be $38/11$ or 3.46.

IN ACTION Using Game Theory to Shape Strategy at General Motors

Game theory often assumes that one player or company must lose for another to win. In the auto industry, car companies typically compete by offering rebates and price cuts. This allows one company to gain market share at the expense of other car companies. Although this win–lose strategy works in the short term, competitors quickly follow the same strategy. The result is lower margins and profitability. Indeed, many customers wait until a rebate or price cut is offered before buying a new car. The short-term win-lose strategy turns into a long-term lose-lose result.

By changing the game itself, it is possible to find strategies that can benefit all competitors. This was the case when General Motors (GM) developed a new credit card that allowed people to apply 5% of their purchases to a new GM vehicle, up to $500 per year with a maximum of $3,500. The credit card program replaced other incentive programs offered by GM. Changing the game helped bring profitability back to GM. In addition, it also helped other car manufacturers who no longer had to compete on price cuts and rebates. In this case, the new game resulted in a win–win situation with GM. Prices, margins, and profitability increased for GM and some of its competitors.

Source: Adam Brandenburger, et al. "The Right Game: Use Game Theory to Shape Strategy," *Harvard Business Review* (July–August 1995): 57.

M4.6 DOMINANCE

The principle of *dominance* can be used to reduce the size of games by eliminating strategies that would never be played. A strategy for a player is said to be dominated if the player can always do as well or better playing another strategy. Any dominated strategy can be eliminated from the game. In other words, a strategy can be eliminated if all its game's outcomes are the same or worse than the corresponding game outcomes of another strategy.

Using the principle of dominance, we reduce the size of the following game:

	Y_1	Y_2
X_1	4	3
X_2	2	20
X_3	1	1

In this game, X_3 will never be played because X can always do better by playing X_1 or X_2. The new game is

	Y_1	Y_2
X_1	4	3
X_2	2	20

Here is another example:

	Y_1	Y_2	Y_3	Y_4
X_1	–5	4	6	–3
X_2	–2	6	2	–20

In this game, Y would never play Y_2 and Y_3 because Y could *always* do better playing Y_1 or Y_4. The new game is

	Y_1	Y_4
X_1	–5	–3
X_2	–2	–20

SUMMARY

Game theory is the study of how optimal strategies are formulated in conflict. Because of the mathematical complexities of game theory, this module is limited to two-person and zero-sum games. A two-person game allows only two people or two groups to be involved in the game. *Zero sum* means that the sum of the losses for one player must equal the sum of the gains for the other player. The overall sum of the losses and gains for both players, in other words, must be zero.

Depending on the actual payoffs in the game and the size of the game, a number of solution techniques can be used. In a pure strategy game, strategies for the players can be obtained without making any calculations. When there is *not* a pure strategy, also called a saddle point, for both players, it is necessary to use other techniques, such as the mixed strategy approach, dominance, and a computer solution for games larger than 2×2.

GLOSSARY

Dominance. A procedure that is used to reduce the size of the game.

Minimax Criterion. A criterion that minimizes one's maximum losses. This is another way of solving a pure strategy game.

Mixed Strategy Game. A game in which the optimal strategy for both players involves playing more than one strategy over time. Each strategy is played a given percentage of the time.

Pure Strategy. A game in which both players will always play just one strategy.

Saddle Point Game. A game that has a pure strategy.

Two-Person Game. A game that has only two players.

Value of the Game. The expected winnings of the game if the game is played a large number of times.

Zero-Sum Game. A game in which the losses for one player equal the gains for the other player.

SOLVED PROBLEMS

Solved Problem M4-1

George Massic (player X) faces the following game. Using dominance, reduce the size of the game if possible.

	Y_1	Y_2
X_1	6	5
X_2	20	23
X_3	15	11

Solution

After carefully analyzing the game, George realizes that he will never play strategy X_1. The best outcome for this strategy (6) is worse than the worst outcome for the other two strategies. In addition, George would never play strategy X_3, for the same reason. Thus, George will always play strategy X_2. Given this situation, player Y would always play strategy Y_1 to minimize her losses. This is a pure strategy game with George playing X_2 and person Y playing strategy Y_1. The value of the game for this problem is the outcome of these two strategies, which is 20.

Solved Problem M4-2

Using the solution procedure for a mixed strategy game, solve the following game:

	Y_1	Y_2
X_1	4	2
X_2	0	10

Solution

This game can be solved by setting up the mixed strategy table and developing the appropriate equations.

		Y_1	Y_2	
		P	$1 - P$	**Expected Value**
X_1	Q	4	2	$\mathbf{4P + 2(1 - P)}$
X_2	$1 - Q$	0	10	$\mathbf{0P + 10(1 - P)}$
Expected Value		$\mathbf{4Q + 0(1 - Q)}$	$\mathbf{2Q + 10(1 - Q)}$	

The equations for Q are

$$4Q + 0(1 - Q) = 2Q + 10(1 - Q)$$
$$4Q = 2Q + 10 - 10Q$$
$$12Q = 10 \text{ or } Q = {}^{10}\!/_{12} \text{ and } 1 - Q = {}^{2}\!/_{12}$$

The equations for P are

$$4P + 2(1 - P) = 0P + 10(1 - P)$$
$$4P + 2 - 2P = 10 - 10P$$
$$12P = 8 \text{ or } P = {}^{8}\!/_{12} \text{ and } 1 - P = {}^{4}\!/_{12}$$

SELF-TEST

- **Before taking the self-test, refer back to the learning objectives at the beginning of the supplement and the glossary at the end of the supplement.**
- **Use the key at the back of the book to correct your answers.**
- **Restudy pages that correspond to any questions that you answered incorrectly or material you feel uncertain about.**

1. In a two-person zero-sum game
   a. each person has two strategies.
   b. whatever is gained by one person is lost by the other.
   c. all payoffs are zero.
   d. a saddle point always exists.
2. A saddle point exists if
   a. the largest payoff in a column is also the smallest payoff in its row.
   b. the smallest payoff in a column is also the largest payoff in its row.
   c. there are only two strategies for each player.
   d. there is a dominated strategy in the game.
3. If the upper and lower values of the game are the same, then
   a. there is no solution to the game.
   b. there is a mixed solution to the game.
   c. a saddle point exists.
   d. there is a dominated strategy in the game.
4. In a mixed strategy game,
   a. each player will always play just one strategy.
   b. there is no saddle point.
   c. each player will try to maximize the maximum of all possible payoffs.
   d. a player will play each of two strategies exactly 50% of the time.
5. In a two-person zero-sum game, it is determined that strategy X_1 dominates strategy X_2. This means
   a. strategy X_1 will never be chosen.
   b. the payoffs for strategy X_1 will be greater than or equal to the payoffs for X_2.
   c. a saddle point exists in the game.
   d. a mixed strategy must be used.
6. In a pure strategy game,
   a. each player will randomly choose the strategy to be used.
   b. each player will always select the same strategy regardless of what the other person does.
   c. there will never be a saddle point.
   d. the value of the game must be computed using probabilities.
7. The solution to a mixed strategy game is based on the assumption that
   a. each player wishes to maximize the long-run average payoff.
   b. both players can be winners with no one experiencing any loss.
   c. players act irrationally.
   d. there is sometimes a better solution than a saddle point solution.

DISCUSSION QUESTIONS AND PROBLEMS

Discussion Questions

M4-1 What is a two-person, zero-sum game?

M4-2 How do you compute the value of the game?

M4-3 What is a pure strategy?

M4-4 Explain the concept of dominance. How is it used?

M4-5 How is a saddle point found in a game?

M4-6 How do you determine whether a game is a pure strategy game or a mixed strategy game?

M4-7 What is a mixed game, and how is it solved?

Problems*

Q • **M4-8** Determine the strategies for X and Y given the following game. What is the value of the game?

	Y_1	Y_2
X_1	2	−4
X_2	6	10

Q • **M4-9** What is the value of the following game and the strategies for A and B?

	B_1	B_2
A_1	19	20
A_2	5	−4

Q : **M4-10** Determine each player's strategy and the value of the game given the following table:

	Y_1	Y_2
X_1	86	42
X_2	36	106

Q : **M4-11** What is the value of the following game?

	S_1	S_2
R_1	21	116
R_2	89	3

* Note: Q means the problem may be solved with QM for Windows.

M4-12 Player A has a \$1 bill and a \$20 bill, and player B has a \$5 bill and a \$10 bill. Each player will select a bill from the other player without knowing what bill the other player selected. If the total of the bills selected is odd, player A gets both of the two bills that were selected, but if the total is even, player B gets both bills.

(a) Develop a payoff table for this game. (Place the sum of both bills in each cell.)
(b) What are the best strategies for each player?
(c) What is the value of the game? Which player would you like to be?

M4-13 Resolve Problem M4-12. If the total of the bills is even, player A gets both of the bills selected, but if the total is odd, player B gets both bills.

M4-14 Solve the following game:

	Y_1	Y_2
X_1	–5	–10
X_2	12	8
X_3	4	12
X_4	–40	–5

M4-15 Shoe Town and Fancy Foot are both vying for more share of the market. If Shoe Town does no advertising, it will not lose any share of the market if Fancy Foot does nothing. It will lose 2% of the market if Fancy Foot invests \$10,000 in advertising, and it will lose 5% of the market if Fancy Foot invests \$20,000 in advertising. On the other hand, if Shoe Town invests \$15,000 in advertising, it will gain 3% of the market if Fancy Foot does nothing; it will gain 1% of the market if Fancy Foot invests \$10,000 in advertising; and it will lose 1% if Fancy Foot invests \$20,000 in advertising.

(a) Develop a payoff table for this problem.
(b) Determine the various strategies using the computer.
(c) How would you determine the value of the game?

M4-16 Assume that a 1% increase in the market means a profit of \$1,000. Resolve Problem M4-15 using monetary value instead of market share.

M4-17 Solve for the optimal strategies and the value of the following game:

A \ B	Strategy B_1	Strategy B_2	Strategy B_3
Strategy A_1	–10	5	15
Strategy A_2	20	2	–20
Strategy A_3	6	2	6
Strategy A_4	–13	–10	44
Strategy A_5	–30	0	45
Strategy A_6	16	–20	6

M4-18 For the following two-person, zero-sum game, are there any dominated strategies? If so, eliminate any dominated strategy and find the value of the game.

		Player Y's Strategies		
		Y_1	Y_2	Y_3
Player X's Strategies	X_1	4	5	10
	X_2	3	4	2
	X_3	8	6	9

M4-19 Refer to Problem M4-8. There is a saddle point in this game, making it a pure strategy game. Ignore this, and solve this as a mixed strategy game. What special condition in the solution indicates that this should not have been solved as a mixed strategy game?

M4-20 Petroleum Research, Inc. (A), and Extraction International, Inc. (B), have both developed a new extraction procedure that will remove metal and other contaminants from used automotive engine oil. The equipment is expensive, and the extraction process is complex, but the approach provides an economical way to recycle used engine oil. Both companies have developed unique technical procedures. Both companies also believe that advertising and promotion are critical to their success. Petroleum Research, with the help of an advertising firm, has developed 15 possible strategies. Extraction International has developed 5 possible advertising strategies. The economic outcome in millions of dollars is shown in the following table. What strategy do you recommend for Petroleum Research? How much money can they expect from their approach?

A \ B	Strategy B_1	Strategy B_2	Strategy B_3	Strategy B_4	Strategy B_5
Strategy A_1	1	2	2	1	4
Strategy A_2	–1	3	–6	7	5
Strategy A_3	10	–3	–5	–20	12
Strategy A_4	6	–8	5	2	2
Strategy A_5	–5	3	3	7	5
Strategy A_6	–1	–1	–3	4	–2
Strategy A_7	–1	0	0	0	–1
Strategy A_8	3	6	–6	8	3
Strategy A_9	2	6	–5	4	–7
Strategy A_{10}	0	0	0	–5	7
Strategy A_{11}	4	8	–5	3	3
Strategy A_{12}	–3	–3	0	3	3
Strategy A_{13}	1	0	0	–2	2
Strategy A_{14}	4	3	3	5	7
Strategy A_{15}	4	–4	4	–5	5

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APPENDIX M4.1: GAME THEORY WITH QM FOR WINDOWS

In this supplement we show how to solve 2 × 2 games using a variety of techniques. In Section M4.5, for example, we discuss how a mixed strategy game could be solved using straightforward algebraic techniques. In this game, player X will receive 4 and 2 by playing strategy X_1 when player Y plays strategies Y_1 and Y_2, respectively. Values of 1 and 10 are the results when player X plays strategy X_2.

To illustrate QM for Windows, let's use these data. Program M4.1 shows the mix that each player should play for each strategy. The value of the game, 3.45, is displayed at the bottom right of the decision table.

PROGRAM M4.1

QM for Windows Output for Game Theory